The Discharge of Gases from a Reservoir through a Pipe

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The well-known Lapple charts for the quick graphical solution to tank discharge problems are modified and corrected.

Consider the flow system of Figure 1. From thermodynamics, the mass velocity through a well-rounded nozzle alone, if we assume ideal isentropic flow, is

$$G = p_0 M_1 \sqrt{\frac{g_c(mw)k}{RT_0}\cdot\frac{1}{Y^{k+1/k-1}}} \quad (1)$$

while the highest flow rate through the nozzle (put $M_1 = 1$) is given by

$$G^* = p_0 \sqrt{\frac{g_c(mw)k}{RT_0}\left(\frac{2}{k+1}\right)^{k+1/k-1}}$$

Through the pipe alone, adiabatic flow is represented by

$$\frac{k+1}{2}\ln\left(\frac{M_2^2 Y_1}{M_1^2 Y_2}\right) - \left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right) + kN = 0 \quad (2)$$

while isothermal flow is represented by

$$k \ln \frac{M_2^2}{M_1^2} - \left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right) + kN = 0 \quad (3)$$

where the Mach number

$$M = \frac{v}{C}$$

the pipe resistance factor

$$N = \frac{4 f_F L}{d}$$

and

$$Y_i = 1 + \frac{k-1}{2} M_i^2$$

The complete derivation of these and related equations can be found in Streeter (1971) and Shapiro (1953).

To find the discharge rate from the reservoir through a pipe requires a trial and error solution of Equation (1) with Equation (2) or (3). Lapple (1943) prepared charts for the quick solution of these equations in terms of the pressure ratio p_3/p_0, the pipe resistance factor N, and the mass velocity ratio G/G_{cni}, where G_{cni} is the maximum possible isothermal flow through a nozzle alone. These charts have been widely reproduced in textbooks and in Perry's Handbook (1950, 1963, 1973).

Now Lapple assumed (1943, p. 408) that the sonic velocity in a gas flowing through a pipe depended on the condition of flow; thus

$$\text{for isothermal flow:} \quad C^2 = g_c RT/(mw) \quad (4)$$

$$\text{for adiabatic flow:} \quad C^2 = k g_c RT/(mw) \quad (5)$$

This is incorrect, for the velocity of propagation of a sonic wave is independent of the type of flow in the pipe, whether isothermal or adiabatic. The term sonic velocity implies a small reversible compression and rarefaction, and this is never isothermal. Thus, the sonic velocity for any flow condition is given by Equation (5), never by Equation (4). In essence, the subtle error sneaks in by assuming that isothermal flow is equivalent in all ways to adiabatic flow of a fictitious gas having $k = 1$ (Lapple, 1943, p. 409; Brown, 1950). This error has been noted previously.

Lapple then used Equation (4) in defining his Mach number, and this in turn led to wrong Mach numbers and errors in some of his derived equations. For example, he finds that the highest velocity attainable at the exit of an isothermal pipe is $M = 1$ instead of $M = k^{-1/2}$. It is interesting that a number of texts also give $M = 1$. However, Streeter (1971) and Shapiro (1953) do have it right.

Figures 2 and 3 are the Lapple charts recalculated and redrawn for the case of an isentropic nozzle followed by adiabatic pipe flow. These charts simply represent the

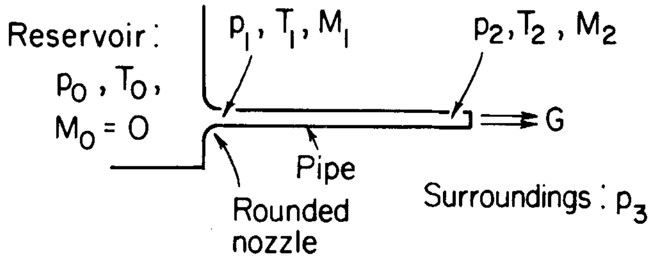

Fig. 1. Pipe discharging from a large chamber.

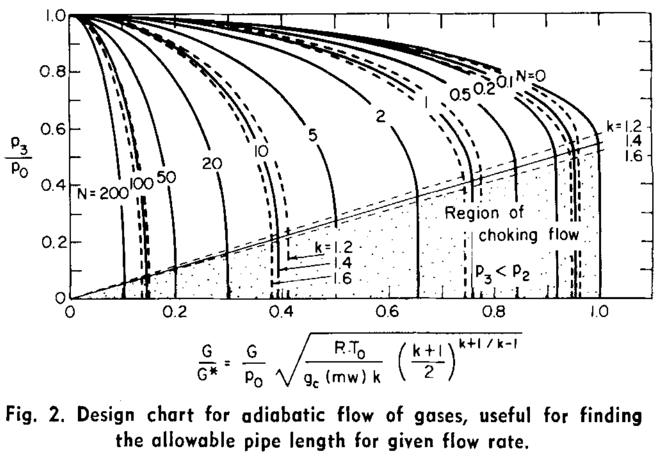

Fig. 2. Design chart for adiabatic flow of gases, useful for finding the allowable pipe length for given flow rate.

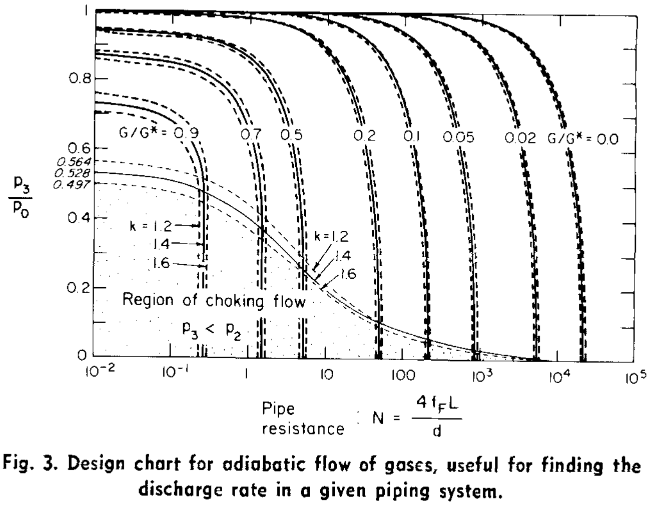

Fig. 3. Design chart for adiabatic flow of gases, useful for finding the discharge rate in a given piping system.

simultaneous solution of Equations (1) and (2), and they can be used to estimate the throughput in a given system or the length of pipe needed for a specified flow rate.

Similar charts for an isentropic nozzle followed by isothermal flow in the pipe can be prepared by combining Equations (1) and (3). However, such charts are not particularly useful because accelerating gas, which remains isothermal during flow, means progressively hotter pipe walls as one moves downstream coupled with increasing heat input to keep the pipe walls at these higher temperatures. Note that stagnant gas at the walls is hotter than the isothermal flowing gas. For the above reason, the adiabatic case is more representative of the isothermal pipe wall. See the discussion by Colburn (1943) following Lapple's paper on this point.

NOTATION

C = sonic velocity, given by Equation (5), m/s
d = pipe diameter, m
f_F = Fanning friction factor
G = mass velocity, kg/s · m^2
G^* = critical mass velocity through an adiabatic nozzle, kg/s · m^2
G_{cni} = critical mass velocity for isothermal flow through a nozzle, kg/s · m^2
g_c = 1 kg · m/s^2 · N, conversion factor
k = C_p/C_v, ratio of specific heats of a gas, k varies between 1.2 and 1.67 for most gases
L = length of pipe
M = v/C, Mach number
(mw) = molecular weight
N = $4 f_F L/d$, pipe resistance factor
p = pressure, N/m^2
R = 8.314 J/mole · K, gas constant
T = temperature, °K
v = velocity of gas, m/s
Y_i = $1 + \frac{k-1}{2} M_i^2$, dimensionless

Subscripts

0 = in the reservoir
1 = at the end of the nozzle and beginning of the pipe
2 = at the end of the pipe
3 = in the surroundings

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A Mixture Theory for Particulate Sedimentation with Diffusivity

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A mixture theory approach has been used by the authors (Bedford and Hill, 1976) to develop equations for the sedimentation under gravity of rigid particles of uniform composition, size, and shape through an incompressible fluid held in a container of constant cross section. It was assumed that the forces acting on the particles included their weight, buoyancy, and a drag force dependent on the particle concentration and particle velocity relative to the fluid.

For sufficiently small particle sizes, the particle diffusivity plays a significant role in sedimentation problems of interest in chemical engineering processes (Fujita, 1962). In the present work, particle diffusivity is introduced into our earlier theory by assuming that the drag force on the particles also depends on the gradient of the particle concentration.

Linearizing the equations in terms of small particle velocities and particle concentration variations, we obtain a generalization of the classical Lamm equation (Fujita, 1962). The linearized equations are hyperbolic in the general case and parabolic for quasi steady sedimentation.

DEVELOPMENT

The conservation of mass equations for the particles and fluid are

$$\frac{\partial \phi_p}{\partial t} + \frac{\partial}{\partial x}(\phi_p U_p) = 0 \quad (1)$$

$$\frac{\partial \phi_f}{\partial t} + \frac{\partial}{\partial x}(\phi_f U_f) = 0 \quad (2)$$

where the volume fractions ϕ_p, ϕ_f satisfy

$$\phi_p + \phi_f = 1 \quad (3)$$

With the sedimentation velocities measured relative to the container, Equations (1) and (2) with Equation